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A computational algorithm for special nth-order pentadiagonal Toeplitz determinants

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Abstract

In this short note, we present a fast and reliable algorithm for evaluating special nth-order pentadiagonal Toeplitz determinants in linear time. The algorithm is suited for implementation using Computer Algebra Systems (CAS) such as MACSYMA and MAPLE. Two illustrative examples are given.

Keywords: Pentadiagonal matrix; Determinants; Algorithm; Computer Algebra Systems (CAS)

1. Introduction and preliminaries

The pentadiagonal Toeplitz linear systems of the form,

$$\begin{bmatrix} a & b & c & 0 & 0 & \cdots & 0 \\ d & a & b & c & 0 & \cdots & 0 \\ e & d & a & b & c & \ddots & \vdots \\ 0 & e & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & d & a & b & c \\ \vdots & \ddots & \ddots & e & d & a & b \\ 0 & \cdots & 0 & 0 & e & d & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ \vdots \\ r_{n-1} \\ r_n \end{bmatrix} \tag{1.1}$$

frequently appear in the solution of fourth-order boundary value problems and elsewhere. For such systems it is generally recommended to check the nonsingularity of the pentadiagonal coefficient matrix before we solve the system. The main objective of this note is to derive a fast computational algorithm for computing

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the nth-order pentadiagonal Toeplitz determinants. In the following section, we are going to derive a new computational symbolic algorithm to compute nth-order pentadiagonal Toeplitz determinants. In Section 3, two examples are given for the sake of illustration.

2. A new computational algorithm

Throughout this section, the parameter λ is just a symbolic name and $\det P$ is the determinant of the pentadiagonal Toeplitz coefficient matrix of the system (1.1).

Following [1–3], we may now formulate the following algorithm:

Algorithm 2.1. To compute $\det P$ we may proceed as follows:

Step 1: Set $c_1 = a$. If $c_1 = 0$ then $c_1 = \lambda$ end if.
Step 2: Compute and simplify $c_2 = a - \frac{bd}{c_1}$.
If $c_2 = 0$ then $c_2 = \lambda$ end if.
Step 3: Set $e_1 = b$ and $f_2 = \frac{d}{c_1}$, then compute and simplify
For k from 3 to n do

$$g_k = \frac{e}{c_{k-2}},$$

$$f_k = \frac{(d - g_k e_{k-2})}{c_{k-1}},$$

$$e_{k-1} = b - cf_{k-1},$$

$$c_k = a - e_{k-1} f_k - cg_k,$$

If $c_k = 0$ then $c_k = \lambda$ end if.

End do.
Step 4: Compute $\det P = \left(\prod_{r=1}^{n} c_r\right)\Big|_{\lambda=0}$.

The algorithm will be referred to as *DETPT*. The cost of the this algorithm is $O(n)$ only. The algorithm is easy to implement using Computer Algebra Systems (CAS) such as MACSYMA and MAPLE.

3. Illustrative examples

In this section, we are going to give two illustrative examples.

Example 3.1. For the matrix A given by

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix},$$

we have

$$(f_2, f_3, f_4, f_5, f_6) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right),$$

$$(g_3, g_4, g_5, g_6) = \left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1\right),$$

$$(e_1, e_2, e_3, e_4, e_5) = \left(1, \frac{1}{2}, \frac{2}{3}, \frac{1}{2}, \frac{1}{2}\right),$$

$$(c_1, c_2, c_3, c_4, c_5, c_6) = \left(2, \frac{3}{2}, \frac{4}{3}, 1, 1, \frac{3}{4}\right).$$

Therefore $\det A = \left(\prod_{r=1}^{6} c_r\right)\Big|_{\lambda=0} = (3)|_{\lambda=0} = 3.$

Example 3.2. For the matrix B given by

$$B = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 \\ 2 & 1 & 1 & 1 & -1 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 & 1 \end{bmatrix},$$

we have

$$(f_2, f_3, f_4, f_5) = \left(1, -\frac{1}{\lambda}, \frac{\lambda - 4}{3\lambda + 2}, \frac{\lambda + 4}{2\lambda + 13}\right),$$

$$(g_3, g_4, g_5) = \left(2, \frac{2}{\lambda}, \frac{2\lambda}{3\lambda + 2}\right),$$

$$(e_1, e_2, e_3, e_4) = \left(1, 2, \frac{\lambda - 1}{\lambda}, \frac{4\lambda - 2}{3\lambda + 2}\right),$$

$$(c_1, c_2, c_3, c_4, c_5) = \left(1, \lambda, \frac{3\lambda + 2}{\lambda}, \frac{2\lambda + 13}{3\lambda + 2}, \frac{2\lambda + 17}{2\lambda + 13}\right).$$

Consequently, $\det B = \left(\prod_{r=1}^{5} c_r\right)\Big|_{\lambda=0} = (2\lambda + 17)|_{\lambda=0} = 17$

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